University of Sask
EE 480.3 Control
Final Examination, April 25, 2002

Note: 3 hour open-book exam. Marks are as indicated at the beginning of each question.
Instructor: K. Takaya

1. (30) An analog system with the transfer function,

$$G(s) = \frac{10}{s^2 + 6s + 5}$$

has state feedback to form a feedback control system as shown in Figure 1. The state variables x_1 and x_2 are digitized at a sampling rate of 10 samples per second.

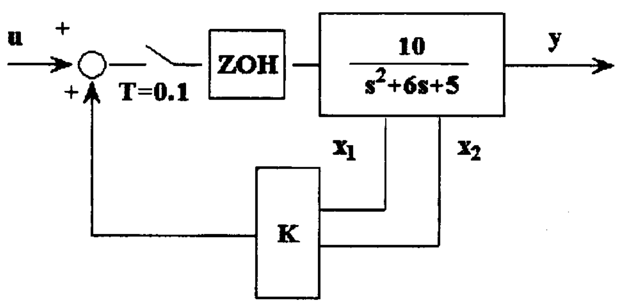

Figure 1: A State Feedback System.

1. Using the state variables defined by

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} y \\ \dot{y} \end{bmatrix},$$

obtain the analog state equation of $G(s)$ which takes the form of

$$\mathbf{x}(t) = A_c\mathbf{x}(t) + \mathbf{b}_c u(t)$$

and the output equation which gives $y(t)$.

2. Obtain the state transition matrix $\Phi_c(t)$ from A_c.
3. Obtain the discrete time state equation expressed in the form of

$$\mathbf{x}(k+1) = A\mathbf{x}(k) + \mathbf{b}u(k)$$

To calculate A and **b**, consider only up to the second order term of the Taylor series expansion,

$$\Phi_c(T) = I + A_c T + A_c^2 \frac{T^2}{2!} + A_c^3 \frac{T^2}{3!} + \cdots$$

4. Write the discrete time transfer function equivalent to the state equation obtained in the previous question. You may either use the previously obtained state equation or calculate directly from $G(s)$.

5. Determine the state feedback gain vector $\mathbf{K} = [K_1, K_2]^T$ that makes the system in Fig. 1 critical damping.

2. (30) A position control system for a communication satellite is shown by the block diagram in Figure 2. A sampling rate of the digital controller is 10 samples per second. The root locus shown in Figure 3 indicates that a phase lead digital compensator,

$$D(z) = K\frac{z - 0.8}{z + 0.2}$$

can bring the control system to satisfactory performance. The discrete transfer function of the satellite dynamics including the zero order hold (ZOH) is known to be

$$G(z) = 0.02\,\frac{T_s^2}{2}\frac{z+1}{(z-1)^2}$$

When the loop gain of $K' = 360 \times 0.02\,\frac{T_s^2}{2} \times K = 0.4$ is set, one of the closed loop pole

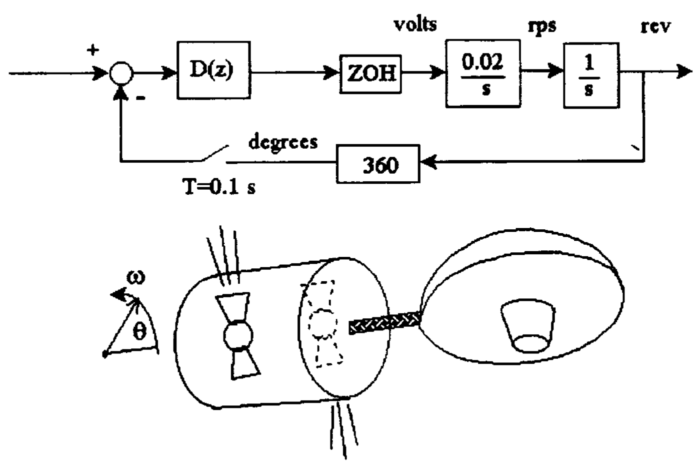

Figure 2: Communication satellite position stabilizer and its block diagram.

is at $z = 0.6$ and other two complex poles are located at $0.4 \pm j0.2$ according to the root locus.

1. Find the damping ratio ζ of the complex conjugate poles.
2. Find the damped natural frequency ω_d of the complex conjugate poles.
3. Calculate the time constant T_{conj} of the complex conjugate poles and the time constant T_{real} of the real pole $z = 0.6$.
4. By three times of trial-and-error, estimate the gain K' that brings the closed loop system to marginally stable (at least one pole on the unit circle).

3. (30) Consider the system shown in Figure 4. The pulse transfer function (step invariant transfer function that includes ZOH) of the plant is given by

$$G(z) = 0.0193\frac{z + 0.9672}{(z-1)(z-0.9048)}.$$

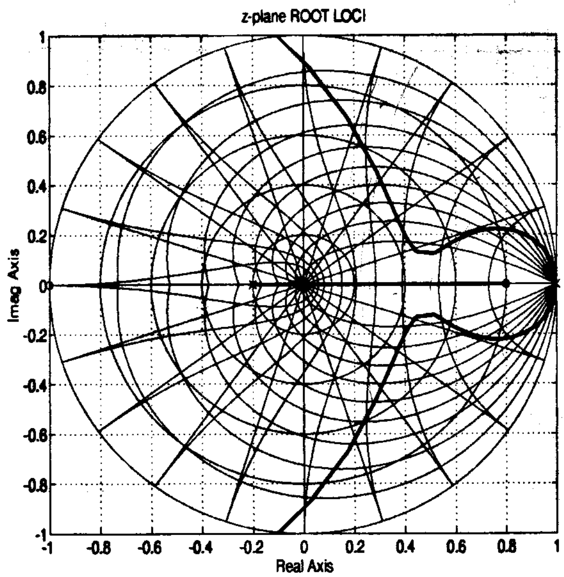

Figure 3: The root locus with a compensator $D(z) = K\frac{z - 0.75}{z + 0.6}$.

Applying the bilinear transformation,

$$w = \frac{2}{T}\frac{z-1}{z+1}$$

to $G(z)$, the frequency response $G(j\omega_w)$ with respect to the warped frequency ω_w is plotted as shown in Figure 5. Design a phase-lead compensator that will improve the phase margin to 60°. In order to satisfy a steady-state constraint for this system, the DC gain of the digital controller must be equal to 1, i.e. $a_0 = 1$ in $D(w)$.

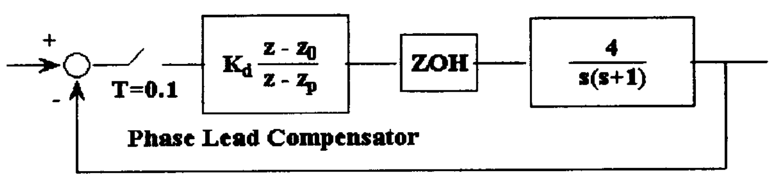

Figure 4: A system compensated by a phase lead controller.

1. What are the gain and phase margin of $G(j\omega_w)$ alone?
2. Choose a new 0dB cross-over frequency ω_{w1} appropriate for phase-lead compensation.
3. For the chosen ω_{w1}, calculate the values of a_1 and b_1 in

$$D(w) = \frac{a_1 w + a_0}{b_1 w + 1}.$$

4. Calculate the values of K_d, z_0 and z_p in the digital compensator transfer function,

$$D(z) = K_d\frac{z - z_0}{z - z_p}.$$

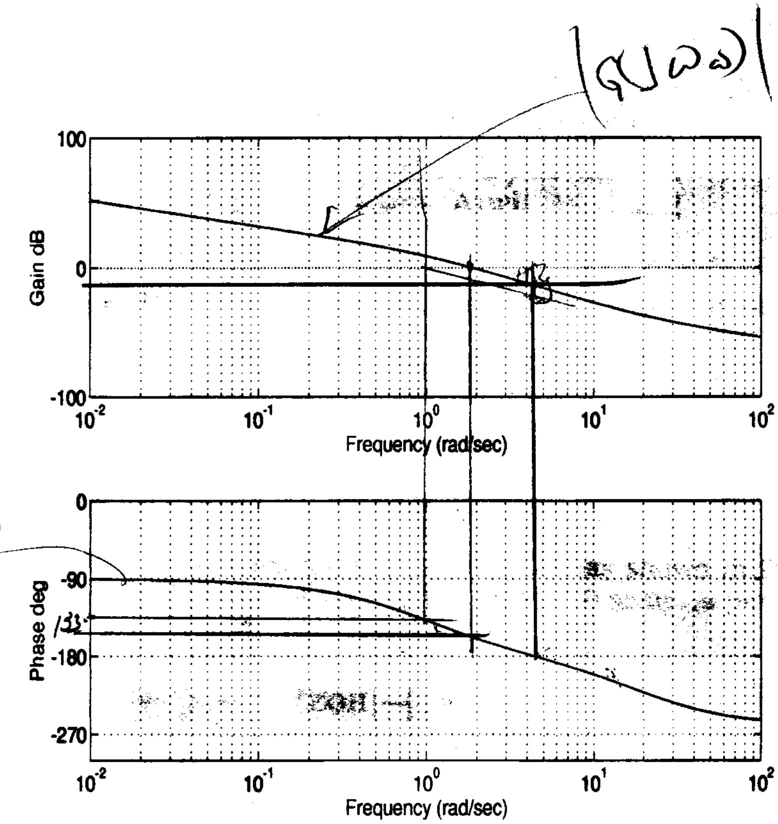

Figure 5: Frequency response of the system plotted with respect to ω_w.

5. A phase-lag compensator can also realize the desired phase margin of 60°. Choose a new 0dB cross-over frequency ω_{w1} appropriate for phase-lag compensation. Then, calculate the values of a_1 and b_1 in $D(w)$ for phase-lag compensation. Note that phase-lag compensator design procedure suggests that $\omega_{w0} = 0.1\omega_{w1}$ is generally a proper choice for $\omega_{w0} = \dfrac{a_0}{a_1}$.

4. (10) A position servo system is shown in the block diagram of Figure 6. This system has a digital compensator $D(z)$ ~~which~~ takes digitized signals of the position $y(t)$ and velocity $v(t)$ and a digital input $u(k)$ for input. K is a velocity feedback gain. Obtain the discrete time transfer function $\dfrac{Y^*(s)}{U^*(s)}$ or $\dfrac{Y(z)}{U(z)}$ in terms of the names of transfer functions, D, G_1, G_2 and K.

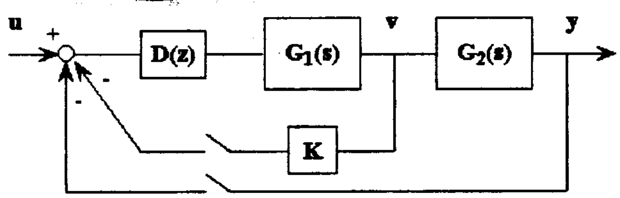

Figure 6: Position and Velocity Feedback Digital Control System.

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